CHAPTER 3

Measurement Uncertainty

Chapter Outline

3.1 Introduction

We have already been introduced to the subject of measurement uncertainty in the last chapter, in the context of defining the accuracy characteristic of a measuring instrument.

Measurement and Instrumentation. http://dx.doi.org/10.1016/B978-0-12-800884-3.00003-4
Copyright © 2016 Elsevier Inc. All rights reserved.

The existence of measurement uncertainty means that we would be entirely wrong to assume (though the uninitiated might assume this) that the output of a measuring instrument or larger measurement system gives the exact value of the measured quantity. Measurement errors are impossible to avoid, although we can minimize their magnitude by good measurement system design accompanied by appropriate analysis and processing of measurement data.

We can divide errors in measurement systems into those that arise during the measurement process and those that arise due to later corruption of the measurement signal by induced noise during transfer of the signal from the point of measurement to some other point. Sections 3.1−3.5 cover the errors occurring during the measurement process, followed by discussion of induced noise in Sections 3.6 and 3.7.

It is extremely important in any measurement system to reduce errors to the minimum possible level and then to quantify the maximum remaining error that may exist in any instrument output reading. However, in many cases, there is a further complication that the final output from a measurement system is calculated by combining together two or more measurements of separate physical variables. In this case, special consideration must also be given to determining how the calculated error levels in each separate measurement should be combined together to give the best estimate of the most likely error magnitude in the calculated output quantity. This subject is considered later in Chapter 4.

The starting point in the quest to reduce the incidence of errors arising during the measurement process is to carry out a detailed analysis of all error sources in the system. Each of these error sources can then be considered in turn, looking for ways of eliminating or at least reducing the magnitude of errors. Errors arising during the measurement process can be divided into two groups, known as systematic errors and random errors.

Systematic errors describe errors in the output readings of a measurement system that are consistently on one side of the correct reading, that is, either all the errors are positive or they are all negative. (NB: Some books use the alternative name bias errors for systematic errors, although this is not entirely correct since systematic errors include errors like sensitivity drift that are not biases.) Two major sources of systematic errors are system disturbance during measurement and the effect of environmental changes (sometimes known as *modifying inputs*), as discussed in Sections 3.4.1 and 3.4.2. Other sources of systematic error include bent meter needles, the use of uncalibrated instruments, drift in instrument characteristics, and poor cabling practices. Even when systematic errors due to the above factors have been reduced or eliminated, some errors remain that are inherent in the manufacture of an instrument. These are quantified by the accuracy value quoted in the published specifications contained in the instrument data sheet.

Random errors, which are also called as *precision errors* in some books, are perturbations of the measurement either side of the true value caused by random and unpredictable effects, such that positive errors and negative errors occur in approximately equal numbers for a series of measurements made of the same quantity. Such perturbations are mainly small, but large perturbations occur from time to time, again unpredictably. Random errors often arise when measurements are taken by human observation of an analog meter, especially where this involves interpolation between scale points. Electrical noise can also be a source of random errors. To a large extent, random errors can be overcome by taking the same measurement a number of times and extracting a value by averaging or other statistical techniques, as discussed later in Chapter 4. However, any quantification of the measurement value and statement of error bounds remain a statistical quantity. Because of the nature of random errors and the fact that large perturbations in the measured quantity occur from time to time, the best that we can do is to express measurements in probabilistic terms: we may be able to assign a 95% or even 99% confidence level that the measurement is a certain value within error bounds of say $\pm 1\%$, but we can never attach a 100% probability to measurement values that are subject to random errors. In other words, even if we say that the maximum error is $\leq \pm 0.5\%$ of the measurement reading, there is still a 1% chance that the error is greater than $\pm 0.5\%$.

Finally, a word must be said about the distinction between systematic and random errors. Error sources in the measurement system must be examined carefully to determine what type of error is present, systematic or random, and to apply the appropriate treatment. In the case of manual data measurements, a human observer may make a different observation at each attempt, but it is often reasonable to assume that the errors are random and that the mean of these readings is likely to be close to the correct value. However, this is only true as long as the human observer is not introducing a parallax-induced systematic error as well by persistently reading the position of a needle against the scale of an analog meter from one side rather than from directly above. A human-induced systematic error is also introduced if an instrument with a first-order characteristic is read before it has settled to its final reading. Wherever a systematic error exists alongside random errors, correction has to be made for the systematic error in the measurements before statistical techniques are applied to reduce the effect of random errors.

3.2 Sources of Systematic Error

The main sources of systematic error in the output of measuring instruments can be summarized as follows:

1. effect of environmental disturbances, often called modifying inputs
2. disturbance of the measured system by the act of measurement
3. changes in characteristics due to wear in instrument components over a period of time
4. resistance of connecting leads

These various sources of systematic error, and ways in which the magnitude of the errors can be reduced, are discussed below.

3.2.1 System Disturbance due to Measurement

Disturbance of the measured system by the act of measurement is a common source of systematic error. If we were to start with a beaker of hot water and wished to measure its temperature with a mercury-in-glass thermometer, then we would take the thermometer, which would initially be at room temperature, and plunge it into the water. In so doing, we would be introducing a relatively cold mass (the thermometer) into the hot water and a heat transfer would take place between the water and the thermometer. This heat transfer would lower the temperature of the water. While the reduction in temperature in this case would be so small as to be undetectable by the limited measurement resolution of such a thermometer, the effect is finite and clearly establishes the principle that, in nearly all measurement situations, the process of measurement disturbs the system and alters the values of the physical quantities being measured.

A particularly important example of this occurs with the orifice plate. This is placed into a fluid-carrying pipe to measure the flow rate, which is a function of the pressure that is measured either side of the orifice plate. This measurement procedure causes a permanent pressure loss in the flowing fluid. The disturbance of the measured system can often be very significant.

Thus, as a general rule, the process of measurement always disturbs the system being measured. The magnitude of the disturbance varies from one measurement system to the next and is affected particularly by the type of instrument used for measurement. Ways of minimizing disturbance of measured systems is an important consideration in instrument design. However, an accurate understanding of the mechanisms of system disturbance is a prerequisite for this.

Measurements in electric circuits

In analyzing system disturbance during measurements in electric circuits, Thévenin's theorem (see Appendix 2) is often of great assistance. For instance, consider the circuit shown in Figure 3.1(a) in which the voltage across resistor R_5 is to be measured by a voltmeter with resistance R_m. Here, R_m acts as a shunt resistance across R_5, decreasing the resistance between points A and B and so disturbing the circuit. Therefore, the voltage E_m measured by the meter is not the value of the voltage E_o that existed prior to measurement. The extent of the disturbance can be assessed by calculating the open-circuit voltage E_o and comparing it with E_m.

Thévenin's theorem allows the circuit of Figure 3.1(a) comprising two voltage sources and five resistors to be replaced by an equivalent circuit containing a single resistance and one

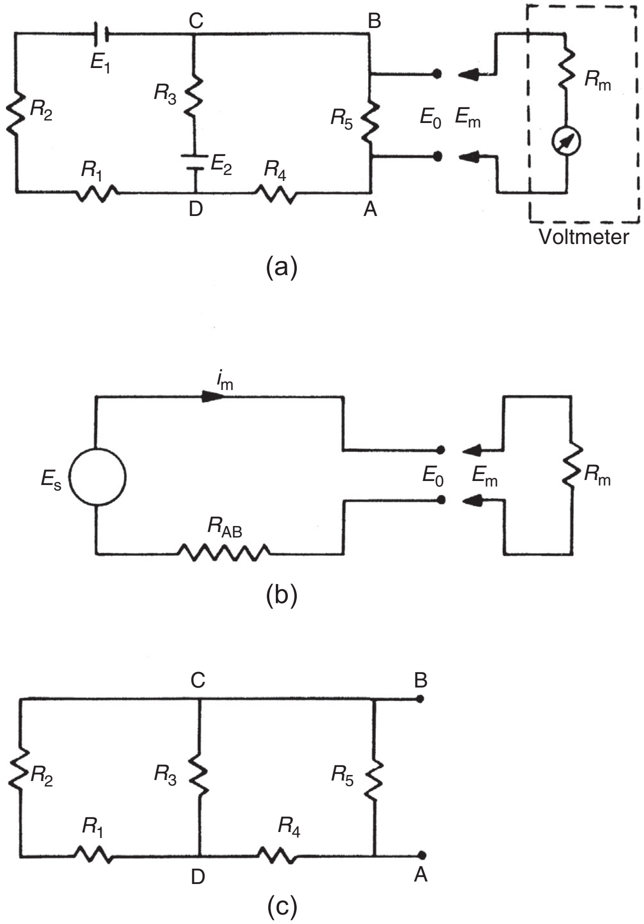

Figure 3.1

Analysis of circuit loading: (a) A circuit in which the voltage across R_5 is to be measured; (b) equivalent circuit by Thévenin's theorem; (c) the circuit used to find the equivalent single resistance R_{AB}.

voltage source, as shown in Figure 3.1(b). For the purpose of defining the equivalent single resistance of a circuit by Thévenin's theorem, all voltage sources are represented just by their internal resistance, which can be approximated to zero, as shown in Figure 3.1(c). Analysis proceeds by calculating the equivalent resistances of sections of the circuit and building these up until the required equivalent resistance of the whole of the circuit is obtained. Starting at C and D, the circuit to the left of C and D consists of a series pair of resistances (R_1 and R_2) in parallel with R_3, and the equivalent resistance can be written as

$$\frac{1}{R_{CD}} = \frac{1}{R_1 + R_2} + \frac{1}{R_3} \quad or \quad R_{CD} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

Moving now to A and B, the circuit to the left consists of a pair of series resistances (R_{CD} and R_4) in parallel with R_5. The equivalent circuit resistance R_{AB} can thus be written as

$$\frac{1}{R_{AB}} = \frac{1}{R_{CD} + R_4} + \frac{1}{R_5} \quad or \quad R_{AB} = \frac{(R_4 + R_{CD})R_5}{R_4 + R_{CD} + R_5}$$

Substituting for R_{CD} using the expression derived previously, we obtain

$$R_{AB} = \frac{\left[\frac{(R_1+R_2)R_3}{R_1+R_2+R_3} + R_4\right]R_5}{\frac{(R_1+R_2)R_3}{R_1+R_2+R_3} + R_4 + R_5} \tag{3.1}$$

Defining I as the current flowing in the circuit when the measuring instrument is connected to it, we can write

$I = \frac{E_o}{R_{AB}+R_m}$, and the voltage measured by the meter is then given by $E_m = \frac{R_m E_o}{R_{AB}+R_m}$.

In the absence of the measuring instrument and its resistance R_m, the voltage across AB would be the equivalent circuit voltage source, whose value is E_o. The effect of measurement is therefore to reduce the voltage across AB by the ratio given by

$$\frac{E_m}{E_o} = \frac{R_m}{R_{AB} + R_m} \tag{3.2}$$

It is thus obvious that as R_m gets larger, the ratio E_m/E_o gets closer to unity, showing that the design strategy should be to make R_m as high as possible to minimize disturbance of the measured system. (Note that we did not calculate the value of E_o, since this is not required in quantifying the effect of R_m.)

Example 3.1

Suppose that the components of the circuit shown in Figure 3.1(a) have the following values: $R_1 = 400\ \Omega$; $R_2 = 600\ \Omega$; $R_3 = 1000\ \Omega$; $R_4 = 500\ \Omega$; $R_5 = 1000\ \Omega$. The voltage across AB is measured by a voltmeter, whose internal resistance is $9500\ \Omega$. What is the measurement error caused by the resistance of the measuring instrument?

Solution

Proceeding by applying Thévenin's theorem to find an equivalent circuit to that of Figure 3.1(a) of the form shown in Figure 3.1(b), and substituting the given component values into the equation for R_{AB} (Eqn (3.1)), we obtain

$R_{AB} = \frac{[(1000^2/2000)+500]1000}{(1000^2/2000)+500+1000} = \frac{1000^2}{2000} = 500\ \Omega$

From Eqn (3.2), we have $\frac{E_m}{E_o} = \frac{R_m}{R_{AB}+R_m}$

The measurement error is given by $(E_o - E_m)$: $E_o - E_m = E_o\left(1 - \frac{R_m}{R_{AB}+R_m}\right)$

Substituting in values $E_o - E_m = E_o\left(1 - \frac{9500}{10000}\right) = 0.95E_o$

Thus, the error in the measured value is 5%.

■

At this point, it is interesting to note the constraints that exist when practical attempts are made to achieve a high internal resistance in the design of a moving-coil voltmeter. Such an instrument consists of a coil carrying a pointer mounted in a fixed magnetic field. As current flows through the coil, the interaction between the field generated and the fixed field causes the pointer it carries to turn in proportion to the applied current (see Chapter 9 for further details). The simplest way of increasing the input impedance (the resistance) of the meter is either to increase the number of turns in the coil or to construct the same number of coil turns with a higher-resistance material. However, either of these solutions decreases the current flowing in the coil, giving less magnetic torque and thus decreasing the measurement sensitivity of the instrument (i.e., for a given applied voltage, we get less deflection of the pointer). This problem can be overcome by changing the spring constant of the restraining springs of the instrument, such that less torque is required to turn the pointer by a given amount. However, this reduces the ruggedness of the instrument and also demands better pivot design to reduce friction. This highlights a very important but tiresome principle in instrument design: any attempt to improve the performance of an instrument in one respect generally decreases the performance in some other aspect. This is an inescapable fact of life with passive instruments such as the type of voltmeter mentioned, and is often the reason for the use of alternative active instruments such as digital voltmeters, where the inclusion of auxiliary power greatly improves performance.

Similar errors due to system loading are also caused when an ammeter is inserted to measure the current flowing in a branch of a circuit. For instance, consider the circuit shown in Figure 3.2(a), in which the current flowing in the branch of the circuit labeled *A-B* is measured by an ammeter with resistance R_m. Here, R_m acts as in series with the resistor R_5 in branch *A-B*, thereby increasing the resistance between points *A* and *B* and so disturbing the circuit. Therefore, the current I_m measured by the meter is not the value of the current I_o that existed prior to measurement. The extent of the disturbance can be assessed by calculating the open-circuit current I_o and comparing it with I_m.

Thévenin's theorem is again a useful tool in analyzing the effect of inserting the ammeter. To apply Thévenin's theorem, the voltage sources are represented just by their internal resistance, which can be approximated to zero as in Figure 3.2(b). This allows the circuit

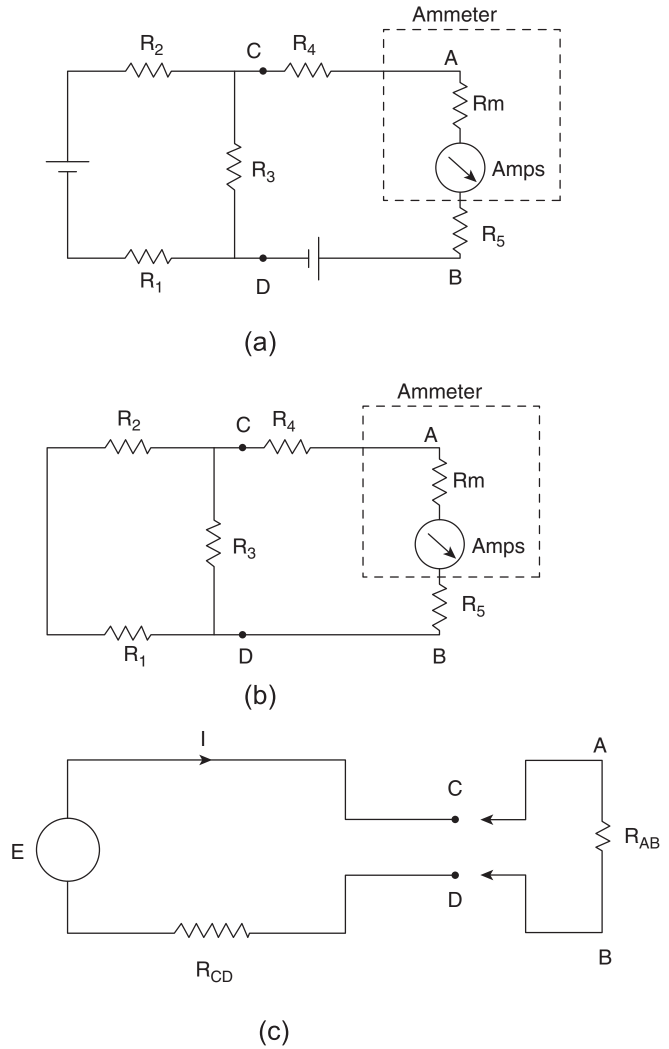

Figure 3.2
Analysis of circuit loading: (a) A circuit in which the current flowing in branch *A-B* of the circuit is to be measured; (b) the circuit with all voltage sources represented by their approximately zero resistance; (c) equivalent circuit by Thévenin's theorem.

of Figure 3.2(a), comprising two voltage sources and five resistors, to be replaced by an equivalent circuit containing just two resistances and a single voltage source, as shown in Figure 3.2(c). Analysis proceeds by calculating the equivalent resistances of sections of the circuit and building these up until the required equivalent resistance of the whole of the circuit is obtained. Starting at C and D, the circuit to the left of C and D consists of a series pair of resistances (R_1 and R_2) in parallel with R_3, and the equivalent resistance can be written as

$$\frac{1}{R_{CD}} = \frac{1}{R_1 + R_2} + \frac{1}{R_3} \quad or \quad R_{CD} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

The current flowing between A and B can be calculated simply by Ohm's law as $I = \frac{E}{R_{AB}+R_{CD}}$

When the ammeter is not in the circuit, $R_{AB} = R_4 + R_5$ and $I = I_0$, where I_0 is the normal (circuit-unloaded) current flowing.

Hence, $I_0 = \dfrac{E}{R_4 + R_5 + \left[\dfrac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}\right]} = \dfrac{E[R_1 + R_2 + R_3]}{[R_4 + R_5][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]}$

With the ammeter in the circuit, $R_{AB} = R_4 + R_5 + R_m$ and $I = I_m$, where I_m is the measured current.

Hence,

$$I_m = \frac{E}{R_4 + R_5 + R_m + \left[\dfrac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}\right]} = \frac{E[R_1 + R_2 + R_3]}{[R_4 + R_5 + R_m][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]}$$

The measurement error is given by the ratio I_m/I_0.

$$\frac{I_m}{I_0} = \frac{[R_4 + R_5][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]}{[R_4 + R_5 + R_m][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]} \tag{3.3}$$

Example 3.2

Suppose that the components of the circuit shown in Figure 3.2(a) have the following values: $R_1 = 250\ \Omega$; $R_2 = 750\ \Omega$; $R_3 = 1000\ \Omega$; $R_4 = 500\ \Omega$; $R_5 = 500\ \Omega$. The current between A and B is measured by an ammeter, whose internal resistance is 50 Ω. What is the measurement error caused by the resistance of the measuring instrument?

Solution

Substituting the parameter values into Eqn (3.3)

$$\frac{I_m}{I_0} = \frac{[R_4 + R_5][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]}{[R_4 + R_5 + R_m][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]}$$
$$= \frac{[1000][2000] + [1000 \times 1000]}{[1050][1000] + [1000 \times 1000]} = \frac{3000}{3100} = 0.968$$

The error is $1 - I_m/I_0 = 1 - 0.968 = 0.032$ or 3.2%.

Thus, the error in the measured current is 3.2%.

Bridge circuits for measuring resistance values are a further example of the need for careful design of the measurement system. The impedance of the instrument measuring the bridge output voltage must be very large in comparison with the component resistances in the bridge circuit. Otherwise, the measuring instrument will load the circuit and draw current from it. This is discussed more elaborately in Chapter 7.

3.2.2 Errors due to Environmental Inputs

An environmental input is defined as an apparently real input to a measurement system that is actually caused by a change in the environmental conditions surrounding the measurement system. The fact that the static and dynamic characteristics specified for measuring instruments are only valid for particular environmental conditions (e.g., of temperature and pressure) has already been discussed at considerable length in Chapter 2. These specified conditions must be reproduced as closely as possible during calibration exercises because, away from the specified calibration conditions, the characteristics of measuring instruments vary to some extent and cause measurement errors. The magnitude of this environment-induced variation is quantified by the two constants known as sensitivity drift and zero drift, both of which are generally included in the published specifications for an instrument. Such variations of environmental conditions away from the calibration conditions are sometimes described as *modifying inputs* to the measurement system because they modify the output of the system. When such modifying inputs are present, it is often difficult to determine how much of the output change in a measurement system is due to a change in the measured variable and how much is due to a change in environmental conditions. This is illustrated by the following example. Suppose we are given a small closed box and told that it may contain either a mouse or a rat. We are also told that the box weighs 0.1 kg when empty. If we put the box onto bathroom scales and observe a reading of 1.0 kg, this does not immediately tell us what is in the box because the reading may be due to one of three things:

1. a 0.9 kg rat in the box (real input)
2. an empty box with a 0.9 kg bias on the scales due to a temperature change (environmental input)
3. a 0.4 kg mouse in the box together with a 0.5 kg bias (real + environmental inputs)

Thus, the magnitude of any environmental input must be measured before the value of the measured quantity (the real input) can be determined from the output reading of an instrument.

In any general measurement situation, it is very difficult to avoid environmental inputs, because it is either impractical or impossible to control the environmental conditions surrounding the measurement system. System designers are therefore charged with the task of either reducing the susceptibility of measuring instruments to environmental inputs or, alternatively, quantifying the effect of environmental inputs and correcting for them in the instrument output reading. The techniques used to deal with environmental inputs and minimize their effect on the final output measurement follow a number of routes as discussed below.

3.2.3 Wear in Instrument Components

Systematic errors can frequently develop over a period of time because of wear in instrument components. Recalibration often provides a full solution to this problem.

3.2.4 Connecting Leads

In connecting together the components of a measurement system, a common source of error is the failure to take proper account of the resistance of connecting leads (or pipes in the case of pneumatically or hydraulically actuated measurement systems). For instance, in typical applications of a resistance thermometer, it is common to find that the thermometer is separated from other parts of the measurement system by perhaps 100 m. The resistance of such a length of 20 gauge copper wire is 7 Ω, and there is a further complication that such wire has a temperature coefficient of 1 mΩ/°C.

Therefore, careful consideration needs to be given to the choice of connecting leads. Not only should they be of adequate cross-section so that their resistance is minimized, but they should be adequately screened if they are thought likely to be subject to electrical or magnetic fields that could otherwise cause induced noise. Where screening is thought essential, then the routing of cables also needs careful planning. In one application in the author's personal experience involving instrumentation of an electric-arc steelmaking furnace, screened signal-carrying cables between transducers on the arc furnace and a control room at the side of the furnace were initially corrupted by high amplitude 50 Hz noise. However, by changing the route of the cables between the transducers and the control room, the magnitude of this induced noise was reduced by a factor of about 10.

3.3 Reduction of Systematic Errors

The prerequisite for the reduction of systematic errors is a complete analysis of the measurement system that identifies all sources of error. Simple faults within a system, such as bent meter needles and poor cabling practices, can usually be readily and cheaply rectified once they have been identified. However, other error sources require more detailed analysis and treatment. Various approaches to error reduction are considered below.

Careful instrument design: Careful instrument design is the most useful weapon in the battle against environmental inputs, by reducing the sensitivity of an instrument to environmental inputs to as low a level as possible. For instance, in the design of strain gauges, the element should be constructed from a material, whose resistance has a very low temperature coefficient (i.e., the variation of the resistance with temperature is very small). However, errors due to the way in which an instrument is designed are not always easy to correct, and a choice often has to be made between the high cost of redesign and the alternative of accepting the reduced measurement accuracy if redesign is not undertaken.

Calibration: Instrument calibration is a very important consideration in measurement systems and therefore calibration procedures are considered in detail in Chapter 5. All instruments suffer drift in their characteristics, and the rate at which this happens depends on many factors, such as the environmental conditions in which instruments are used and the frequency of their use. The error due to an instrument being out-of-calibration is never zero, even immediately after the instrument has been calibrated, because there is always some inherent error in the reference instrument that a working instrument is calibrated against during the calibration exercise. Nevertheless, the error immediately after calibration is of low magnitude. The calibration error then grows steadily with the drift in instrument characteristics until the time of the next calibration. The maximum error that exists just before an instrument is recalibrated can therefore be made smaller by increasing the frequency of recalibration so that the amount of drift between calibrations is reduced.

Method of opposing inputs: The method of opposing inputs compensates for the effect of an environmental input in a measurement system by introducing an equal and opposite environmental input that cancels it out. One example of how this technique is applied is in the type of millivoltmeter shown in Figure 3.3. This consists of a coil suspended in a fixed magnetic field produced by a permanent magnet. When an unknown voltage is applied to the coil, the magnetic field due to the current interacts with the fixed field and causes the coil (and a pointer attached to the coil) to turn. If the coil resistance R_{coil} is sensitive to temperature, then any environmental input to the system in the form of a temperature change will alter the value of the coil current for a given applied voltage and so alter the pointer output reading. Compensation for this is made by introducing a compensating resistance R_{comp} into the circuit, where R_{comp} has a temperature coefficient that is equal in magnitude but opposite in sign to that of the coil. Thus, in response to an increase in temperature, R_{coil} increases but R_{comp} decreases, and so the total resistance remains approximately the same.

High-gain feedback: The benefit of adding high-gain feedback to many measurement systems is illustrated by considering the case of the voltage-measuring instrument, whose block diagram is shown in Figure 3.4. In this system, the unknown voltage E_i is applied to a coil of torque constant K_c, and the induced torque turns a pointer against the

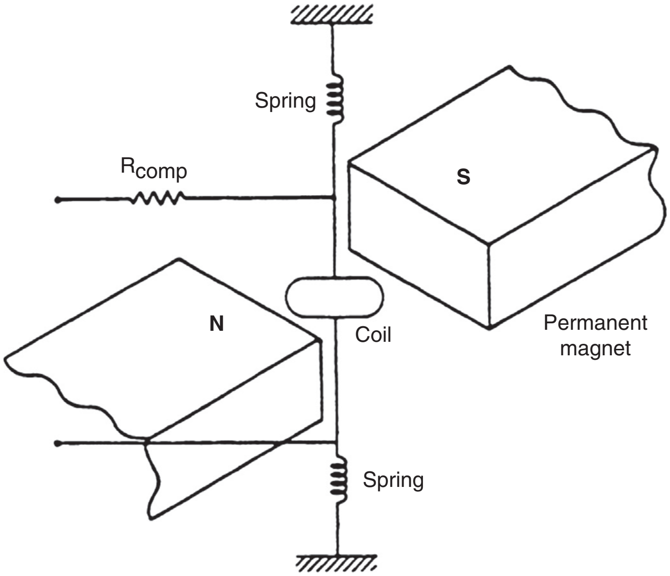

Figure 3.3
Analog millivoltmeter mechanism.

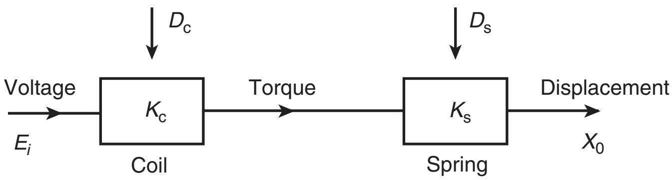

Figure 3.4
Block diagram for voltage-measuring instrument.

restraining action of a spring with spring constant K_s. The effect of environmental inputs on the coil and spring constants is represented by variables D_c and D_s. In the absence of environmental inputs, the displacement of the pointer X_o is given by $X_o = K_c K_s E_i$. However, in the presence of environmental inputs, both K_c and K_s change, and the relationship between X_o and E_i can be affected greatly. Therefore, it becomes difficult or impossible to calculate E_i from the measured value of X_o. Consider now what happens if the system is converted into a high-gain, closed-loop one, as shown in Figure 3.5, by adding an amplifier of gain constant K_a and a feedback device with gain constant K_f. Assume also that the effect of environmental inputs on the values of K_a and K_f are represented by D_a and D_f. The feedback device feeds back a voltage E_o proportional to the pointer displacement X_o. This is compared with the unknown voltage E_i by a

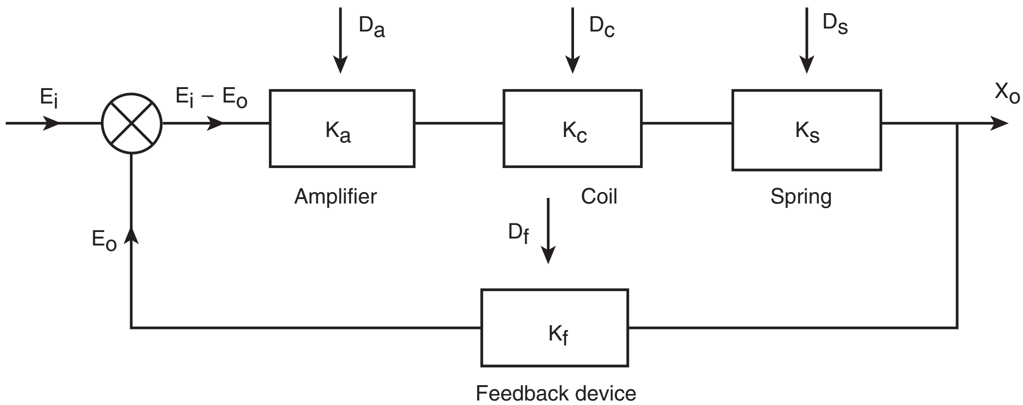

Figure 3.5
Block diagram of voltage-measuring instrument with high-gain feedback.

comparator and the error is amplified. Writing down the equations of the system, we have

$$E_o = K_f X_o; \quad X_o = (E_i - E_o)K_a K_c K_s = (E_i - K_f X_o)K_a K_c K_s$$

$$\text{Thus,} \quad E_i K_a K_c K_s = (1 + K_f \cdot K_a \cdot K_c \cdot K_s)X_o \quad \text{i.e.} \quad X_o = \frac{K_a K_c K_s}{1 + K_f \cdot K_a \cdot K_c \cdot K_s} E_i \tag{3.4}$$

Because K_a is very large (it is a high-gain amplifier), $K_f \cdot K_a \cdot K_c \cdot K_s \gg 1$, and Eqn (3.4) reduces to

$$X_o = E_i / K_f$$

This is a highly important result because we have reduced the relationship between X_o and E_i to one that involves only K_f. The sensitivity of the gain constants K_a, K_c, and K_s to the environmental inputs D_a, D_c, and D_s has thereby been rendered irrelevant, and we only have to be concerned with one environmental input D_f. Conveniently, it is usually easy to design a feedback device that is insensitive to environmental inputs; this is much easier than trying to make a motor or spring insensitive. Thus, high-gain feedback techniques are often a very effective way of reducing a measurement system's sensitivity to environmental inputs. However, one potential problem that must be mentioned is that there is a possibility that high-gain feedback will cause instability in the system. Therefore, any application of this method must include careful stability analysis of the system.

Signal filtering: One frequent problem in measurement systems is corruption of the output reading by periodic noise, often at a frequency of 50 Hz caused by pickup through the close proximity of the measurement system to apparatus or current-carrying cables operating on a mains supply. Periodic noise corruption at higher frequencies is also often introduced by mechanical oscillation or vibration within some component of a measurement system. The amplitude of all such noise components can be substantially attenuated by the inclusion of filtering of an appropriate form in the system, as discussed at greater length in Chapters 6 and 12. Band-stop filters can be especially useful where corruption is of one particular known frequency, or, more generally, low-pass filters are employed to attenuate all noise in the frequency range of 50 Hz and above. Measurement systems with a low-level output, such as a bridge circuit measuring a strain-gauge resistance, are particularly prone to noise, and Figure 3.6 shows the typical corruption of a bridge output by 50 Hz pickup. The beneficial effect of putting a simple passive RC low-pass filter across the output is shown in Figure 3.6.

Manual correction of output reading: In the case of errors that are due either to system disturbance during the act of measurement or to environmental changes, a good measurement technician can substantially reduce errors at the output of a measurement system by calculating the effect of such systematic errors and making appropriate correction to the instrument readings. This is not necessarily an easy task, and requires all disturbances in the measurement system to be quantified. This procedure is carried out automatically by intelligent instruments.

Intelligent instruments: Intelligent instruments contain extra sensors that measure the value of environmental inputs and automatically compensate the value of the output reading. They have the ability to deal very effectively with systematic errors in

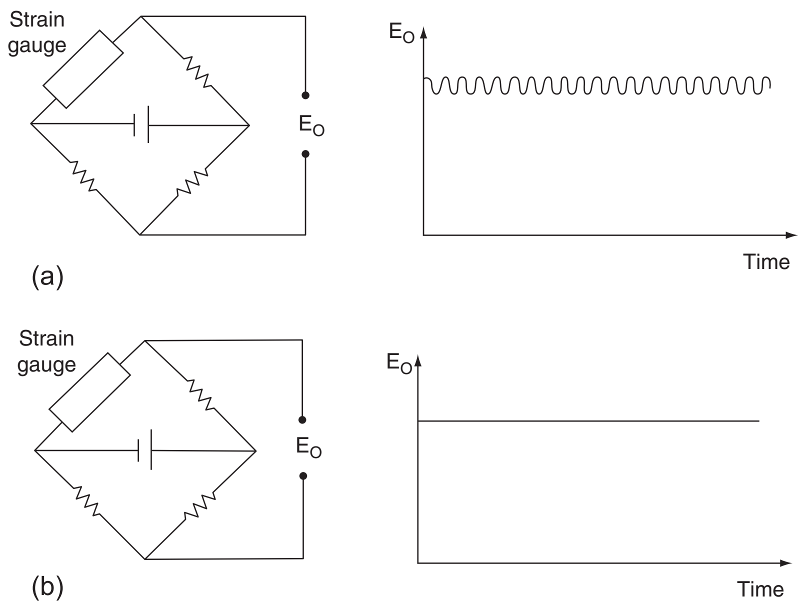

Figure 3.6
Signal filtering. (a) Typical corruption of bridge circuit output by 50Hz pickup; (b) use of low pass filter to remove noise from bridge circuit output.

measurement systems, and errors can be attenuated to very low levels in many cases. A more detailed discussion of intelligent instruments can be found in Chapter 10.

3.4 Quantification of Systematic Errors

Once all practical steps have been taken to eliminate or reduce the magnitude of systematic errors, the final action required is to estimate the maximum remaining error that may exist in a measurement due to systematic errors. This quantification of the maximum likely systematic error in a measurement requires careful analysis.

3.4.1 Quantification of Individual Systematic Error Components

The first complication in the quantification of systematic errors is that it is not usually possible to specify an exact value for a component of systematic error, and the quantification has to be in terms of a "best estimate." Once systematic errors have been reduced as far as reasonably possible using the techniques explained in Section 3.3, a sensible approach to estimate the various kinds of remaining systematic error would be as follows.

Environmental condition errors

If a measurement is subject to unpredictable environmental conditions, the usual course of action is to assume midpoint environmental conditions and specify the maximum measurement error as $\pm x\%$ of the output reading to allow for the maximum expected deviation in environmental conditions away from this midpoint. Of course, this only refers to the case where the environmental conditions remain essentially constant during a period of measurement but vary unpredictably on perhaps a daily basis. If random fluctuations occur over a short period of time from causes like random drafts of hot or cold air, this is a random error rather than a systematic error that has to be quantified according to the techniques explained later in Chapter 4.

Calibration errors

All measuring instruments suffer from drift in their characteristics over a period of time. The schedule for recalibration is set so that the frequency at which an instrument is calibrated means that the drift in characteristics by the time just before the instrument is due for recalibration is kept within an acceptable limit. The maximum error just before the instrument is due for recalibration becomes the basis for estimating the maximum likely error. This error due to the instrument being out-of-calibration is a usually in the form of a bias. The best way to express this is to assume some midpoint value of calibration error and compensate all measurements by this midpoint error. The maximum measurement error over the full period of time between when the instrument has just been calibrated and time just before the next calibration is due can then be expressed as $\pm x\%$ of the output reading.

Example 3.3

The recalibration frequency of a pressure transducer with a range of 0–10 bar is set so that it is recalibrated once the measurement error has grown to +1% of the full-scale reading. How can its inaccuracy be expressed in the form of a $\pm x\%$ error in the output reading?

Solution

Just before the instrument is due for recalibration, the measurement error will have grown to +0.1 bar (1% of 10 bar). An amount of half this maximum error, i.e., 0.05 bar should be subtracted from all measurements. Having done this, the error just after the instrument has been calibrated will be −0.05 bar (−0.5% of full-scale reading) and, the error just before the next recalibration will be +0.05 bar (+0.5% of full-scale reading). The inaccuracy due to calibration error can then be expressed as ±0.5% of full-scale reading.

System disturbance errors

Disturbance of the measured system by the act of measurement itself introduces a systematic error that can be quantified for any given set of measurement conditions. However, if the quantity being measured and/or the conditions of measurement can vary, the best approach is to calculate the maximum likely error under worst-case system loading and then to express the likely error as a plus or minus value of half this calculated maximum error, as suggested for calibration errors.

Measurement system loading errors

These have a similar effect to system disturbance errors and are expressed in the form of $\pm x\%$ of the output reading, where x is half the magnitude of the maximum predicted error under the most adverse loading conditions expected.

3.4.2 Calculation of Overall Systematic Error

The second complication in the analysis to quantify systematic errors in a measurement system is the fact that the total systemic error in a measurement is often composed of several separate components, for example, measurement system loading, environmental factors, and calibration errors. A worst-case prediction of maximum error would be to simply add up each separate systematic error. For example, if there are three components of systematic error with a magnitude of ±1% each, a worst-case prediction error would be the sum of the separate errors, that is, ±3%. However, it is very unlikely that all components of error would be at their maximum or minimum values simultaneously. The usual course of action is therefore to combine separate sources of systematic error using a *root-sum-squares method*. Applying this method for n systematic component errors of magnitude $\pm x_1\%$, $\pm x_2\%$, $\pm x_3\%$,...$\pm x_n\%$, the best prediction of likely maximum systematic error by the root-sum-squares method is

$$\text{Error} = \pm\sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots\ldots + x_n^2}$$

Example 3.4

Three separate sources of systematic error are identified in a measurement system and, after reducing the magnitude of these errors as much as possible, the magnitudes of the three errors are estimated to be as follows:

System loading: ±1.2%

Environmental changes: 0.8%

Calibration error: 0.5%

Calculate the maximum possible total systematic error and the likely system error by the root-mean-squares method.

Solution

The maximum possible system error is $\pm(1.2 + 0.8 + 0.5)\% = \pm 2.5\%$

Applying the root-mean-squares method,

$$\text{likely error} = \pm\sqrt{1.2^2 + 0.8^2 + 0.5^2} = \pm 1.53\%$$

Before closing this discussion on quantifying systematic errors, a word of warning must be given about the use of manufacturers' data sheets. When instrument manufacturers provide data sheets with an instrument that they have made, the measurement uncertainty or inaccuracy value quoted in the data sheets is the best estimate that the manufacturer can give about the way that the instrument will perform when it is new, used under specified conditions and recalibrated at the recommended frequency. Therefore, this can only be a starting point in estimating the measurement accuracy that will be achieved when the instrument is actually used. Many sources of systematic error may apply in a particular measurement situation that are not included in the accuracy calculation in the manufacturer's data sheet, and careful quantification and analysis of all systematic errors are necessary, as described above.

3.5 Sources and Treatment of Random Errors

Random errors in measurements are caused by unpredictable variations in the measurement system. In some books, they are known by the alternative name *precision errors*. Typical sources of random error are as follows:

1. measurements are taken by human observation of an analog meter, especially where this involves interpolation between scale points
2. electrical noise
3. random environmental changes, for example, sudden draft of air

Random errors are usually observed as small perturbations of the measurement either side of the correct value, that is, positive errors and negative errors occur in approximately equal numbers for a series of measurements made of the same constant quantity. Therefore, random errors can largely be eliminated by calculating the average of a number of repeated measurements. Of course, this is only possible if the quantity being measured remains at a constant value during the repeated measurements. This averaging process of repeated measurements can be done automatically by intelligent instruments, as discussed in Chapter 10.

While the process of averaging over a large number of measurements substantially reduces the magnitude of random errors, it would be entirely incorrect to assume that this totally eliminates random errors. This is because the mean of a number of measurements would only be equal to the correct value of the measured quantity if the measurement set contained an infinite number of values. In practice, it is impossible to take an infinite number of measurements. Therefore, in any practical situation, the process of averaging over a finite number of measurements only reduces the magnitude of random error to a small (but nonzero) value. The degree of confidence that the calculated mean value is close to the correct value of the measured quantity can be indicated by calculating the standard deviation or variance of the data, these being parameters that describe how the measurements are distributed about the mean value (see Section 4.3). This leads on to a more formal quantification of this degree of confidence in terms of the standard error of the mean (see Section 4.7).

3.6 Induced Measurement Noise

The earlier sections in this chapter have already provided a detailed analysis of error sources that arise during the measurement process of sensing the value of a physical variable and generating an output signal. However, further errors are often created in measurement systems when electrical signals from measurement sensors and transducers are corrupted by induced noise. This induced noise arises both within the measurement circuit itself and also during the transmission of measurement signals to remote points. The aim when designing measurement systems is always to reduce such induced noise voltage levels as far as possible. However, it is usually not possible to eliminate all such noise, and signal processing has to be applied to deal with any noise that remains.

Noise voltages can exist either in serial mode or common mode forms. Serial mode noise voltages act in series with the output voltage from a measurement sensor or transducer, which can cause very significant errors in the output measurement signal. The extent to which series mode noise corrupts measurement signals is measured by a quantity known as the *signal-to-noise ratio*. This is defined as

$$\text{Signal-to-noise ratio} = 20\log_{10}\left(\frac{V_s}{V_n}\right)$$

where V_s is the mean voltage level of the signal and V_n is the mean voltage level of the noise. In the case of AC noise voltages, the root-mean-squared value is used as the mean.

Common-mode noise voltages are less serious, because they cause the potential of both sides of a signal circuit to be raised by the same level, and thus the level of the output measurement signal is unchanged. However, common-mode voltages do have to be considered carefully, since they can be converted into series mode voltages in certain circumstances.

Noise can be generated from sources both external and internal to the measurement system. Induced noise from external sources arises in measurement systems for a number of reasons that include their proximity to mains-powered equipment and cables (causing noise at the mains frequency), proximity to fluorescent lighting circuits (causing noise at twice the mains frequency), proximity to equipment operating at audio and radio frequencies (causing noise at corresponding frequency), switching of nearby DC and AC circuits, and corona discharge (both of the latter causing induced spikes and transients). Internal noise includes thermoelectric potentials, shot noise and potentials due to electrochemical action.

3.6.1 Inductive Coupling

The primary mechanism by which external devices such as mains cables and equipment, fluorescent lighting and circuits operating at audio or radio frequencies generate noise is

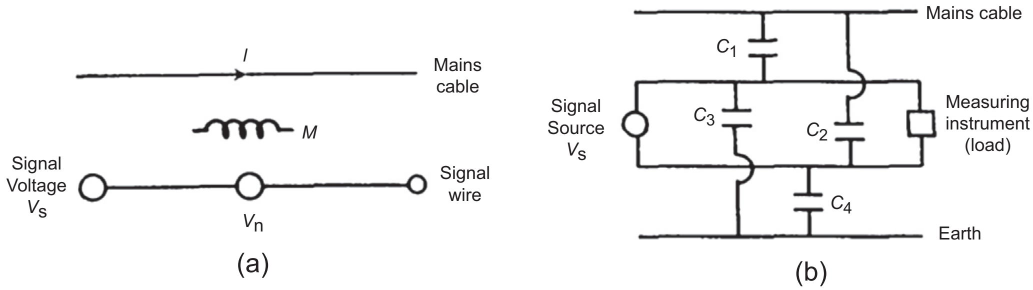

Figure 3.7
Noise induced by coupling: (a) Inductive coupling; (b) capacitive (electrostatic) coupling.

through inductive coupling. If signal-carrying cables are close to such external cables or equipment, a significant mutual inductance M can exist between them, as shown in Figure 3.7(a), and this can generate a series mode noise voltage of several millivolts given by $V_n = M\dot{I}$, where $\dot{I}$ is the rate of change of current in the mains circuit.

3.6.2 Capacitive (Electrostatic) Coupling

Capacitive coupling, also known as electrostatic coupling, can also occur between the signal wires in a measurement circuit and a nearby mains-carrying conductor. The magnitude of the capacitance between each signal wire and the mains conductor is represented by the quantities C_1 and C_2 in Figure 3.7(b). In addition to these capacitances, a capacitance can also exist between the signal wires and earth, represented by C_3 and C_4 in the figure. It can be shown (Cook, 1979)[1] that the series mode noise voltage V_n is zero if the coupling capacitances are perfectly balanced, that is, if $C_1 = C_2$ and $C_3 = C_4$. However, exact balance is unlikely in practice, since the signal wires are not perfectly straight, causing the distances and thus the capacitances to the mains cable and to earth to vary. Thus, some series-mode noise voltage induced by capacitive coupling usually exists.

3.6.3 Noise due to Multiple Earths

As far as possible, measurement signal circuits are isolated from earth. However, leakage paths often exist between measurement circuit signal wires and earth at both the source (sensor) end of the circuit and also the load (measuring instrument) end. This does not cause a problem as long as the earth potential at both ends is the same. However, it is common to find that other machinery and equipment carrying large currents are connected to the same earth plane. This can cause the potential to vary between different points on the earth plane. This situation, which is known as *multiple earths*, can cause a series mode noise voltage in the measurement circuit.

[1] Cook. B.J. (1979) Journal of Measurement and Control, 12 (8), pp. 326–335.

3.6.4 Noise in the Form of Voltage Transients

When motors and other electrical equipment (both AC and DC) are switched on and off, large changes of power consumption suddenly occur in the electricity supply system. This can cause voltage transients ("spikes") in measurement circuits connected to the same power supply. Such noise voltages are of large magnitude but short time duration. *Corona discharge* can also cause voltage transients on the mains power supply. This occurs when the air in the vicinity of high voltage DC circuits becomes ionized and discharges to earth at random times.

3.6.5 Thermoelectric Potentials

Whenever metals of two different types are connected together, a thermoelectric potential (sometimes called a *thermal emf*) is generated according to the temperature of the joint. This is known as the *thermoelectric effect* and is the physical principle on which temperature-measuring thermocouples operate (see Chapter 14). Such thermoelectric potentials are only a few millivolts in magnitude and so the effect is only significant when typical voltage output signals of a measurement system are of a similar low magnitude.

One such situation is where one emf-measuring instrument is used to monitor the output of several thermocouples measuring the temperatures at different points in a process control system. This requires a means of automatically switching the output of each thermocouple to the measuring instrument in turn. Nickel–iron reed-relays with copper connecting leads are commonly used to provide this switching function. This introduces a thermocouple effect of magnitude 40 μV/°C between the reed-relay and the copper connecting leads. There is no problem if both ends of the reed-relay are at the same temperature because then the thermoelectric potentials will be equal and opposite and so cancel out. However, there are several recorded instances where, because of lack of awareness of the problem, poor design has resulted in the two ends of a reed-relay being at different temperatures and causing a net thermoelectric potential. The serious error that this introduces is clear. For a temperature difference between the two ends of only 2 °C, the thermoelectric potential is 80 μV, which is very large compared with a typical thermocouple output level of 400 μV.

Another example of the difficulties that thermoelectric potentials can create becomes apparent in considering the following problem that was reported in a current-measuring system. This system had been designed such that the current flowing in a particular part of a circuit was calculated by applying it to an accurately calibrated wire-wound resistance of value 100 Ω and measuring the voltage drop across the resistance. In calibration of the system, a known current of 20 μA was applied to the resistance and a voltage of 2.20 mV was measured by an accurate high-impedance instrument. Simple application of Ohm's law reveals that such a voltage reading indicates a current value of 22 μA. What then was the explanation for this discrepancy? The answer once again is a thermoelectric potential. Because the designer was not aware of thermoelectric potentials, the circuit had been constructed such that one side of the standard resistance was close to a power transistor, creating a difference in temperature between the two ends of the resistor of 2 °C. The thermoelectric potential associated with this was sufficient to account for the 10% measurement error found.

3.6.6 Shot Noise

Shot noise occurs in transistors, integrated circuits, and other semiconductor devices. It consists of random fluctuations in the rate of transfer of carriers across junctions within such devices.

3.6.7 Electrochemical Potentials

These are potentials that arise within measurement systems due to electrochemical action. Poorly soldered joints are a common source.

3.7 Techniques for Reducing Induced Measurement Noise

Prevention is always better than cure, and much can be done to reduce the level of measurement noise by taking appropriate steps when designing the measurement system.

3.7.1 Location and Design of Signal Wires

Both the mutual inductance and capacitance between signal wires and other cables are inversely proportional to the square of the distance between the wires and the cable. Thus, noise due to inductive and capacitive coupling can be minimized by ensuring that signal wires are positioned as far away as possible from such noise sources. A minimum separation of 0.3 m is essential, and a separation of at least 1 m is preferable. Noise due to inductive coupling is also substantially reduced if each pair of signal wires is twisted together along its length. This design is known as a *twisted pair*, and is illustrated in Figure 3.8. In the first loop, wire A is closest to the noise source and has a voltage V_1 induced in it, while wire B has an induced noise voltage V_2. For loop 2, wire B is closest to the noise source and has an induced voltage V_1 while wire A has an induced voltage V_2. Thus the total voltage induced in wire A is $V_1 + V_2$ and in wire B it is $V_2 + V_1$ over these two loops. This pattern continues for all the loops and hence the two wires have an identical voltage induced in them.

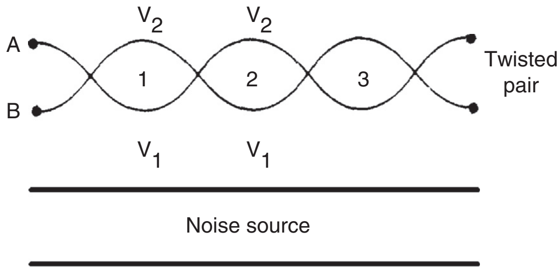

Figure 3.8
Cancellation of induced noise.

3.7.2 Earthing

Noise due to multiple earths can be avoided by good earthing practices. In particular, this means keeping earths for signal wires and earths for high-current equipment entirely separate. Recommended practice is to install four completely isolated earth circuits as follows:

Power earth: provides a path for fault currents due to power faults.

Logic earth: provides a common line for all logic circuit potentials.

Analog earth (ground): provides a common reference for all analog signals.

Safety earth: connected to all metal parts of equipment to protect personnel should power lines come into contact with metal enclosures.

3.7.3 Shielding

Shielding consists of enclosing the signal wires in an earthed, metal shield that is itself isolated electrically from the signal wires. The shield should be earthed at only one point, preferably the signal source end. A shield consisting of braided metal eliminates 85% of noise due to capacitive coupling while a lapped metal foil shield eliminates noise almost entirely. The wires inside such a shield are normally formed as a twisted pair so that protection is also provided against induced noise due to nearby electromagnetic fields. Metal conduit is also sometimes used to provide shielding from capacitive-coupled noise, but the necessary supports for the conduit provide multiple earth points and lead to the problem of earth loops.

3.7.4 Other Techniques

Despite taking the precautions outlined above when designing a measurement system, some induced noise is often unavoidable. Several techniques are available for dealing with this.

The *phase-locked loop* is often used as a signal-processing element to clean up poor quality signals. Although this is primarily a circuit for measuring the frequency of a signal, as described in Chapter 7, it is also useful for noise removal because its output waveform is a pure (i.e., perfectly clean) square wave at the same frequency as the input signal, irrespective of the amount of noise, modulation, or distortion on the input signal.

Lock-in amplifiers are also commonly used to extract DC or slowly varying measurement signals from noise. The input measurement signal is modulated into a square wave AC signal, whose amplitude varies with the level of the input signal. This is normally achieved by either a relay or a field-effect transistor. As a relay is subject to wear, the transistor is better. An alternative method is to use an analog multiplier. Also, in the case of optical signals, the square wave can be produced by chopping the measurement signals using a set of windows in a rotating disk. This technique is frequently used with transducers like photodiodes that often generate large quantities of noise.

3.8 Summary

This chapter has introduced the subject of measurement uncertainty. We have learned that measurement errors are a fact of life and, although we can do much to reduce the magnitude of errors, we can never reduce errors entirely to zero. We also learned that errors occur both during the measurement process and also during transmission of measurement signals from the point of measurement to some other point through induced noise. We started the chapter off by noting that uncertainty during the measurement process comes in two distinct forms, known respectively as *systematic errors* and *random errors*. We learned that the nature of systematic errors was such that the effect on a measurement reading was to make it either consistently greater than or consistently less than the true value of the measured quantity. Random errors on the other hand are entirely random in nature, such that successive measurements of a constant quantity are randomly both greater than and less than the true value of the measured quantity.

In our subsequent study of systematic measurement errors, we first examined all the sources of this kind of error. Following this, we looked at all the techniques that are available for reducing the magnitude of systematic errors arising from the various error sources identified. Finally, we examined ways of quantifying the remaining systemic measurement error after all reasonable means of reducing error magnitude had been applied. We also briefly discussed the nature of random errors and the use of averaging over a number of measurements to reduce their effect. However, since the detailed analysis of random errors is a lengthy subject, this was deferred until Chapter 4.

This chapter then went on consider the additional measurement errors that are generated when electrical signals from measurement sensors and transducers are corrupted by induced noise during transmission of the measurement signal from the point of measurement to some other point. We examined ways of reducing induced noise voltage levels as far as possible but noted that it is usually not possible to eliminate all such noise, and that signal processing has to be applied to deal with any noise that remains.

3.9 Problems

3.1 Explain the difference between systematic and random errors. What are the typical sources of these two types of errors?

3.2 In what ways can the act of measurement cause a disturbance in the system being measured?

3.3 In the circuit shown in Figure 3.9, the resistor values are given by $R_1 = 1000\ \Omega$; $R_2 = 1000\ \Omega$; $V = 20$ V. The voltage across AB (i.e., across R_2) is measured by a voltmeter, whose internal resistance is given by $R_m = 9500\ \Omega$.

(i) What will be the reading on the voltmeter?

(ii) What would the voltage across AB be if the voltmeter was not loading the circuit (i.e., if $R_m =$ infinity)?

(iii) What is the measurement error due to the loading effect of the voltmeter?

3.4 Suppose that the components in the circuit shown in Figure 3.1(a) have the following values:

$$R_1 = 330\ \Omega;\quad R_2 = 1000\ \Omega;\quad R_3 = 1200\ \Omega;\quad R_4 = 220\ \Omega;\quad R_5 = 270\ \Omega.$$

If the instrument measuring the output voltage across AB has a resistance of $5000\ \Omega$, what is the measurement error caused by the loading effect of this instrument?

3.5 (a) Why does a fully calibrated voltmeter never give the correct value when measuring the voltage in an electrical circuit (assume that it is used at the same temperature that it was calibrated at)?

(b) What steps can be taken to reduce the measurement error?

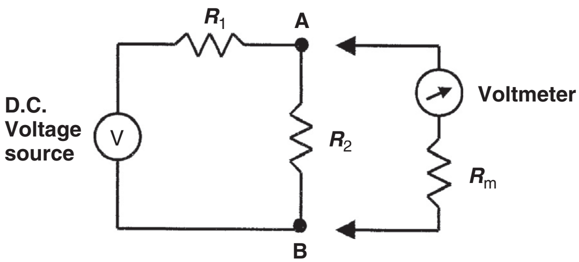

Figure 3.9
Circuit for problems 3.3, 3.5, and 3.12.

(c) In the circuit shown in Figure 3.9, the resistor values are given by $R_1 = 500\ \Omega$; $R_2 = 500\ \Omega$; $V = 12$ V. The voltage across AB (i.e., across R_2) is measured by a voltmeter whose internal resistance is given by $R_m = 7500\ \Omega$.

(i) What will be the reading on the voltmeter?

(ii) What would the voltage across AB be if the voltmeter was not loading the circuit (i.e., if R_m = infinity)?

(iii) What is the measurement error due to the loading effect of the voltmeter?

3.6 (a) Explain what is meant by the term "modifying inputs."

(b) Explain briefly what measures can be taken to reduce or eliminate the effect of modifying inputs.

3.7 (a) Explain what a "thermoelectric potential" is. Discuss the circumstances in which thermoelectric potentials do and do not cause problems in electrical circuits.

(b) Why does a voltmeter load an electric circuit when it is placed into it to measure a voltage value and what is the effect of this loading?

(c) Suppose that the components in the circuit shown in Figure 3.1(a) have the following values:

$$R_1 = 470\ \Omega;\quad R_2 = 1200\ \Omega;\quad R_3 = 1600\ \Omega;\quad R_4 = 330\ \Omega;\quad R_5 = 180\ \Omega.$$

If the instrument measuring the output voltage across AB has a resistance of 10,000 Ω, what is the measurement error caused by the loading effect of this instrument?

3.8 Instruments are normally calibrated and their characteristics defined for particular standard ambient conditions. What procedures are normally taken to avoid measurement errors when using instruments that are subjected to changing ambient conditions?

3.9 The voltage across a resistance R_5 in the circuit of Figure 3.10 is to be measured by a voltmeter connected across it.

(a) If the voltmeter has an internal resistance (R_m) of 4750 Ω, what is the measurement error?

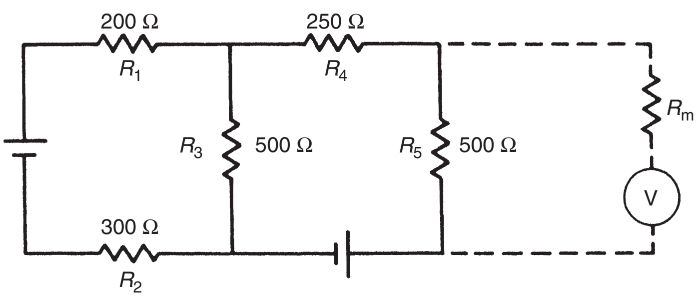

Figure 3.10
Circuit for problems 3.9 and 3.11.

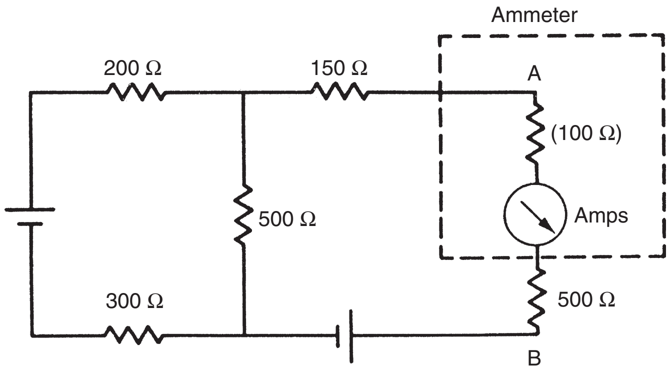

Figure 3.11
Circuit for problems 3.10 and 3.13.

(b) What value would the voltmeter internal resistance need to be in order to reduce the measurement error to 1%?

3.10 In the circuit shown in Figure 3.11, the current flowing between A and B is measured by an ammeter, whose internal resistance is 100 Ω. What is the measurement error caused by the resistance of the measuring instrument?

3.11 (a) Why is there an error in the measured voltage when a voltmeter is inserted into an electrical circuit to measure the voltage across a component in the circuit?

(b) The voltage across a resistance R_5 in the circuit of Figure 3.10 is to be measured by a voltmeter connected across it.

(i) If the voltmeter has an internal resistance (R_m) of 8950 Ω, what is the measurement error?

(ii) What value would the voltmeter internal resistance need to be in order to reduce the measurement error to 0.5%?

3.12 (a) Explain why a voltmeter never reads exactly the correct value when it is applied to an electrical circuit to measure the voltage between two points.

(b) For the circuit shown in Figure 3.9, show that the voltage E_m measured across points A and B by the voltmeter is related to the true voltage E_o by the following expression:

$$\frac{E_m}{E_o} = \frac{R_m(R_1 + R_2)}{R_1(R_2 + R_m) + R_2R_m}$$

(c) If the parameters in Figure 3.9 have the following values, $R_1 = 500\ \Omega$; $R_2 = 500\ \Omega$; $R_m = 4750\ \Omega$, calculate the percentage error in the voltage value measured across points AB by the voltmeter.

3.13 (a) Why is there a measurement error when an ammeter is inserted to measure the current in a branch of a circuit?

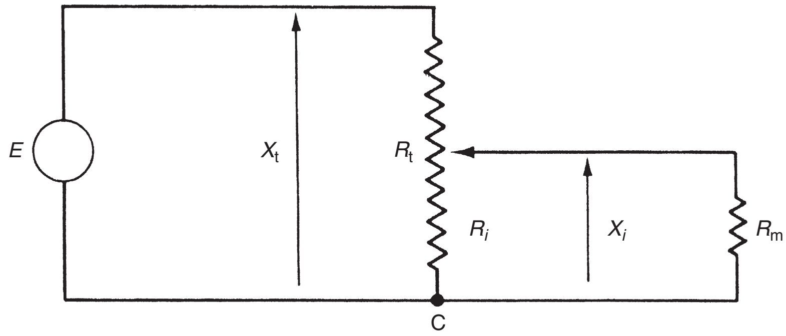

Figure 3.12
Circuit for problem 3.14.

(b) In the circuit shown in Figure 3.2, the current flowing between A and B is measured by an ammeter, whose internal resistance is 85 Ω. The resistance values are given by

$$R_1 = 300\ \Omega; \quad R_2 = 200\ \Omega; \quad R_3 = 500\ \Omega; \quad R_4 = 150\ \Omega; \quad R_5 = 500\ \Omega.$$

What is the measurement error caused by the insertion of the measuring instrument into the circuit?

3.14 The output of a potentiometer is measured by a voltmeter having a resistance R_m, as shown in Figure 3.12. R_t is the resistance of the total length X_t of the potentiometer and R_i is the resistance between the wiper and common point C for a general wiper position X_i. Show that the measurement error due to the resistance R_m of the measuring instrument is given by

$$\text{Error} = E\frac{R_i^2(R_t - R_i)}{R_t\left(R_iR_t + R_mR_t - R_i^2\right)}$$

Hence show that the maximum error occurs when X_i is approximately equal to $2X_t/3$.

(Hint—differentiate the error expression with respect to R_i and set to 0. Note that maximum error does not occur exactly at $X_i = 2X_t/3$, but this value is very close to the position where the maximum error occurs).